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ABSTRACT

The second in a four-part sequence for the student seeking Advanced Placement, this booklet emphasizes theory and applications of the derivative and the definite integral. Overall goals and performance objectives are speficied. A course outline, teaching suggestions, and references to state-adopted texts are given for each topic covered. A sample test is included. (DT)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

CALCULUS 2

5297.42

MATHEMATICS

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

ED 067286

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

CALCULUS 2
5297.42

(EXPERIMENTAL)

Written By

Maria Gomez

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The second of a 4-quin sequence leading to the CEEB Advanced Placement AB Test in May. Emphasizes theory and applications of the derivative and definite integral.

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GOALS

I. The student should develop an understanding of the concept of derivative and acquire the skills necessary to find the derivative of any algebraic function.

II. The student should be able to apply the concept of a derivative to the solution of problems related to curve tracing, to related rates of change, and to the velocity and acceleration of a particle in straight line motion.

III. The student should understand the concept of definite integral and its relation to the area under a curve.

KEY TO REFERENCES
(*State Adopted)

Jon — Johnson, R.E. and Kiokemeister, F. L. Calculus with Analytic Geometry. Boston: Allyn and Bacon, Inc., 1965.

*PM-C(1) — Protter, Murray H. and Morrey, Charles B., Jr. Calculus with Analytic Geometry: A First Course. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1970.

*Tom — Thomas, George B., Jr. Elements of Calculus and Analytic Geometry. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1967.

*BI-BII — Babrow, Daniel G. Introductory Calculus, Parts 1 and 2. Chicago: Encyclopedia Britannica Educational Corp., 1961.

PERFORMANCE OBJECTIVES FOR GOALS

The student will be able to:

I. A. Find the derivative of polynomial functions by applying the definition of the derivative.

B. 1. Find the slope of the tangent and normal, at a given point of the graph of a polynomial function, applying the definition of the derivative.

2. Find the equations of the tangent and normal line to the graph of any polynomial function, at a given point.

C. Find continuity or discontinuity of a function. Prove the answer.

D. 1. Differentiate functions which are given as sums, differences, products or quotients of two polynomial functions.

2. Differentiate the product of three or more functions applying the associative property for multiplication and the derivative formula for a product.

3. Differentiate any algebraic function using the formulas for differentiation of products, quotients, roots, and composition of functions by applying the Chain Rule.

4. Apply the Chain Rule to differentiate functions given in parametric form.

5. Differentiate functions given in implicit form.

6. Find second and third order derivatives of functions given in explicit or implicit form.

II. A. 1. Determine the intervals where a given function is increasing or decreasing using the first derivative test.

2. Find the absolute or relative maxima and minima of a given function, applying the first derivative test.

3. Find the intervals where a given function is concave upward or concave downward by applying the second derivative test.

4. Find the points of inflection of a given function.

5. Find the extrema of a given function by using the second derivative test.

6. Apply Rolle's Theorem, or The Mean Value Theorem, in the solution of problems.

B. 1. Apply the theory of extrema to a given function in order to sketch the graph of the function.
2. Apply the theory of extrema to solve problems related to maximum area, maximum volume or minimum cost.
3. Given the position function of a particle moving on a straight line, find its velocity and acceleration at any time t.
4. Given the position function of a particle moving on a straight line, find the instant (t) at which the velocity and/or acceleration has a given value.
5. Discuss the motion of a point moving on a straight line if the position function is given.
6. Find the instantaneous rate of change of a variable, of a given function, at a specified time (t).

III. A. 1. Calculate the approximate area under a curve by using inscribed or circumscribed rectangles.
2. Compute the area under a curve as a limit.

B. List the properties of the definite integral.

C. 1. Evaluate definite integrals involving powers of x.
2. Evaluate definite integrals by making a change of variable and changing limits.

D. 1. Verify the Intermediate Value Theorem for a given function at a given point.
2. Verify the Theorem of the Mean for Integrals for a given definite integral, by finding the appropriate number c in the given interval.

SEQUENCE, SOURCE, AND STRATEGIES

COURSE OUTLINE

I. The Derivative

A. Definition of derivative

B. Geometric Interpretation

C. Continuity of a Differentiable Function

D. Differentiation Formula

E. Notation

SUGGESTED STRATEGIES

A. Definition of derivative

B. Geometric interpretation of derivative

C. 1. Prove the theorem "If a function is differentiable, then it is continuous."
2. Show, using the absolute value function, that the converse of the theorem is not true.

D. Using the definition of derivative, develop the differentiation formulas for:

1. Constant function: $f(x) = k$
2. Linear function: $f(x) = x$
3. Power function: $f(x) = x^n$
4. $F(x) = k\, f(x)$
5. Sum of two functions: $F(x) = f(x) + g(x)$
6. Product ot two functions: $F(x) = f(x) \cdot g(x)$
7. Quotient of two functions: $F(x) = \frac{f(x)}{g(x)}$
8. Chain rule for composite functions: $F(x) = f(g(x))$

9. Chain rule for functions given in parametric form: $y = f(t)$, $x = g(t)$
10. Implicit differentiation
11. Higher order derivatives

E. Different kinds of notation: $f^1(x)$, $\frac{dy}{dx}$, y^1 etc.

PAGE NUMBERS OF SOURCES

	Jon	PM-C(1)	Tom	B 1	B 2
A	99-101	77- 79	67- 72	170	
B	101-105	79- 83	64- 66	116	
C	105-107	112-113	145-153	None	
D	107-119	126-146	109-145	183	
E	119-123	None	70- 71	173	

COURSE OUTLINE

II. Applications of the Derivative

A. Theory of the extrema of a function

1. Extreme Value Theorem
2. Increasing and decreasing functions: definition
3. Relationship between the first derivative of a function and the property of being increasing or decreasing.
4. Conditions for the first derivative of a function to be an extremum
5. Rolle's Theorem
6. Mean Value Theorem

B. Application of the theory of extrema

1. Curve sketching
2. Problems for maximum area, volume, minimum cost, etc.
3. Velocity and acceleration of a particle moving along a straight line
4. Related rates of change

SUGGESTED STRATEGIES

A.
1. "Extreme Value Theorem" Show the validity intuitively.
2. Definition of increasing and decreasing functions at a point
3. Prove the theorem that relates the first derivative of a function and the property of being increasing or decreasing at a point.
4. Prove the theorem that states the conditions of the first derivative of a function, at a point c, in order for f(c) to be an extremum.
5. Prove Rolle's theorem.
6. Prove the Mean Value Theorem.
7. Relative extrema of a function: First derivative test.
8. Definition of concavity of a function
9. Test for concavity of a function: Second derivative test.
10. Define point of inflection.
11. Prove the second derivative test for maximum or minimum of a function.

PAGE NUMBERS AND SOURCES

		Jon	PM-C(1)	Tom	B 1	B 2
A	1.	128	147			None
	2.	128	160	176		7
	3.	129	158	161		7
	4.	129	149, 159	177		11
	5.	129-130	153	191		42
	6.	131-132	154	193		43
	7.	136-139	160	161		22
	8.	140-141	163-165	168-170		10
	9.	141-142	165-166	169		None
	10.	142	166	170		9
	11.	146	165	168		24
B	1.	146	167	171-176		
	2.	147-154	171-182	179-190		33
	3.	154-159	83- 88	72- 77	116, 195	
	4.	159-160	193-199	163-168		52

COURSE OUTLINE

III. The Definite Integral

A. Definition of definite integral as limit of a sum

B. Properties of the definite integral

1. $\int_a^b cf(x)dx = c\int_a^b f(x)\ dx$

2. $\int_a^b \left[f(x) + g(x)\right] dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. If f(x) is integrable in $[a,b]$ then it is bounded there.

C. Fundamental Theorem of Calculus

D. Intermediate Value Theorems for Calculus

1. Intermediate Value Theorem
2. Theorem of the Mean for Integrals

E. Integration Formulas

1. Power integration formula
2. Integration by change of variable

SUGGESTED STRATEGIES

A. 1. a. Calculate the approximate area under a curve between two vertical lines by using a specific number of inscribed rectangles, and then the same number of circumscribed rectangles.

b. Repeat the process increasing the number of rectangles.

c. Compare the results as the number of rectangles increases.

2. Definition of integrable function over the closed interval $[a,b]$.

3. Prove theorems related to the integrability of functions:

a. If $f(x)$ is increasing or decreasing in an interval $a \leq x \leq b$, then it is integrable there.

b. If $f(x)$ is continuous in $[a,b]$ then it is integrable on $[a,b]$.

B. Prove:

1. $$\int_a^b c\, f(x)\, dx = c \int_a^b f(x)\, dx$$

2. $$\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

3. If $f(x)$ is integrable in $[a,b]$ then it is bounded there.

4. If $f(x)$ is integrable in $[a,b]$ and m and M are numbers such that $m \leq f(x) \leq M$ for $a \leq x \leq b$ then

$$m(b-a) \leq \int_a^b f(x)\, dx \leq M(b-a)$$

5. If f and g are integrable in the interval $[a,b]$ and $f(x) \leq g(x)$ for each x in $[a,b]$ then

$$\int_a^b f(x)\, dx \leq \int_a^b g(x)\, dx$$

6. If $f(x)$ is continuous in $[a,b]$ and c is a number in $[a,b]$ then

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$$

C. Prove the Fundamental Theorem of Calculus.

D. 1. Prove the Intermediate Value Theorem.

2. Prove the Theorem of the Mean for Integrals.

E. Integration Formulas

1. Prove the power integration formula.
2. Integration of composite functions by change of variable.

PAGE NUMBERS AND SOURCES

		Jon	PM-C(1)	Tom	B 1	B 2
A	1.	175-188	200-212	231-241		61
	2.	188-189	212-216	265		64
	3.	184	214	None		None
	4.	189-192	216-219	265		72
B	1.	193	219	None		70
	2.	193	219	None		None
	3.	192	220	265		None
	4.	194	220	None		None
	5.	193	221	None		None
	6.	193	221	265		77
C	1.	195	224-228	248-256		81
D	1.	198	228-230	242		None
	2.	199	230-234	266		None
E	1.	200-203	227-228	205		85
	2.	203-206	235-238	209		87

SAMPLE TEST ITEMS

I. A.1 1. Given $f(x) = x^2 - 3x + 2$, find $f^1(-2)$ using the definition of derivative.

B.1 2. Find the equations of tangent line and normal line to the graph of $f(x) = 1 - (x + 1)^{1/3}$ at the point $[-1, f(-1)]$ by applying the definition of derivative.

B. 2 3. The equation of the tangent to the curve $y = x^2 - 4x$ at the point where the curve intercepts the y - axis is:

a. $y = 8x - 4$ b. $y = -4x$ c. $y = -4$
d. $y = 4x$ e. $y = 4x - 8$

C.1 4. If f(x) is continuous at the point where x = a, which of the following statements may be false?

a. $\lim_{x \to a} f(x)$ exists b. $\lim_{x \to a} f(x) = f(a)$

c. f(a) exists d. f(a) is defined.

e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

D.1 5. If $y = \frac{2 - x}{3x + 1}$ then $\frac{dy}{dx} =$

a. $-\frac{7}{(3x + 1)^2}$ b. $\frac{6x - 5}{(3x + 1)^2}$ c. $-\frac{9}{(3x + 1)^2}$

d. $\frac{7}{(3x + 1)^2}$ e. $\frac{7 - 6x}{(3x + 1)^2}$

D. 2 6. Given $y = (4x + 1)^2 (1 - x)^3$ then $y^1 =$

a. $(4x + 1)^2 (1-x)^2 (5-20x)$
b. $(4x + 1) (1 - x)^2 (4x + 11)$
c. $5(4x + 1) (1 - x)^2 (1-4x)$
d. $(4x + 1) (1 - x)^2 (11 - 20x)$
e. $-24 (4 x 1) (1 - x)^2$

D.3 7. If $y = \frac{x}{\sqrt{1 - x^2}}$; $\frac{dy}{dx} =$

a. $\frac{1 - 2x^2}{(1 - x^2)^{3/2}}$ b. $\frac{1}{1 - x^2}$ c. $\frac{1}{\sqrt{1 - x^2}}$

d. $\frac{1 - 2x^2}{(1 - x^2)^{1/2}}$ e. None of these

D.4 8. If $y = f(x^2)$ and $f^1(x) = \sqrt{5x - 1}$, then $\frac{dy}{dx} =$

a. $2x\sqrt{5x^2 - 1}$ b. $\sqrt{5x - 1}$ c. $2x\sqrt{5x - 1}$

d. $\frac{\sqrt{5x - 1}}{2x}$ e. None of these

9. If $y = \sqrt{x^2 + 1}$, then the derivative of y^2 with respect to x^2 is:

a. 1 b. $\frac{x^2 + 1}{2x}$ c. $\frac{x}{2(x^2 + 1)}$

d. $\frac{2}{x}$ e. $\frac{x^2}{x^2 + 1}$

10. If $x = 2t - 1$, and $y = 3 - 4t^2$ then $\frac{dy}{dx}$ is

a. $4t$ b. $-4t$ c. $-\frac{1}{4t}$

d. $2(x + 1)$ e. $-4(x + 1)$

D.5 11. If y is a differentiable function of x, and $x^3 - xy + y^3 = 1$, then $\frac{dy}{dx}$ is equal to:

a. $\frac{3x^2}{x - 3y^2}$ b. $\frac{3x^2 - 1}{1 - 3y^2}$

c. $\frac{y - 3x^2}{3y^2 - x}$ d. $\frac{3x^2 - 3y^2 - y}{x}$

e. $\frac{3x^2 + 3y^2}{x}$

D.6 12. If $f(x) = x^4 - 4x^2$, then $f^4(2)$ equals

a. 48 b. 0 c. 24 d. 144 e. 16

II. A.1 13. Given $f(x) = 5 - x^{2/3}$, find $f^1(x)$ and determine the intervals where the function increases or decreases.

A.2 14. If, for all x, $f^1(x) = (x - 2)^4 (x - 1)^3$, it follows that the function f has:

a. a relative minimum at $x = 1$
b. a relative maximum at $x = 1$
c. both a relative minimum at $x = 1$, and a relative maximum at $x = 2$
d. relative minimum at $x = 1$ and at $x = 2$.

A.3 15. The curve $y = \frac{1 - x}{x + 3}$ is concave up when:

a. $x > 3$ b. $1 < x < 3$ c. $x > 1$
d. $x < 1$ e. $-x < 3$

A.4 16. The number of inflection points of $f(x) = x^4 - 4x^2$ is

a. 0 b. 1 c. 2 d. 3 e. 4

A.5 17. The total number of relative maximum and minimum points of the function whose derivative, for all x, is given by $f^1(x) = x(x - 3)^2 (x + 1)^4$ is

a. 0 b. 1 c. 2 d. 3 e. None of these.

A.6 18. If c represents the number defined by Rolle's Theorem, then for the function $f(x) = x^3 - 3x^2$ on the interval $0 \leq x \leq 3$, c is equal to:

a. 2 b. 1 c. 0 d. $\sqrt{2}$ e. None of these.

B.1 19. Sketch the graph of $y = x^4 - 6x^2$, showing any maximum, minimum or inflection points.

B.2 20. A line is drawn through the point (1, 2) forming a right triangle with the position x- and y- axes. The slope of the line forming the triangle of smallest area is:

a. -1 b. -2 c. -4 d. $-\frac{1}{2}$ e. -3

B.3 21. The motion of a particle on a straight line is given by a = $a = t^3 - 6t^2 + 12t - 8$. The acceleration is positive.

a. when $t > 2$
b. for all t, $t = 2$
c. when $t < 2$
d. for all $1 < t < 3$
e. for $1 < t < 2$

B.4 22. A particle moves along a horizontal line according to the law $s = t^4 - 6t^3 + 12t^2 + 3$. The particle is at rest when t is equal to:

a. 1 or 2 b. 0 c. $\frac{9}{4}$ d. 0, 2, or 3

e. None of these

B.5 23. If a particle moves along a line according to the law $s = t^5 + 5t^4$, then the number of times it reverses direction is:

a. 0 b. 1 c. 2 d. 3 e. 4

B.6 24. A vertical circular cylinder has radius r feet and height h feet. If the height and radius both increase at the constant rate of 2 feet/second, then the rate in feet^2/second at which the lateral surface area increases is:

a. $4\pi r$ b. $2\pi (r-h)$ c. $4\pi (r + h)$ d. $4\pi rh$
e. $4\pi h$

III. A.1 25. Find the area of the region bounded by $y = x^2$, the line $x = 1$, the line $x = 2$ and the x - axis by

a. inscribing
b. circumscribing 4 rectangles of length $\frac{1}{4}$

B.1 26. If f(x) is continuous on the closed interval $[a, b]$ and k is a constant, then $\int_a^b k\ f(x)\,dx$ is equal to:

a. $k\ (b - a)$ b. $k\ [f(b) - f(a)]$

c. $K\ F(b - a)$ where $\frac{d\ F\ (x)}{dx} = f\ (x)$

d. $k \int_a^b f\ (x)\,dx$ e. $\frac{k\ [f\ (x)]^{\ 2}}{2}\Big]_a^b$

C.1 27. $\int_{-1}^{1} (x^2 - x - 1)\,dx =$

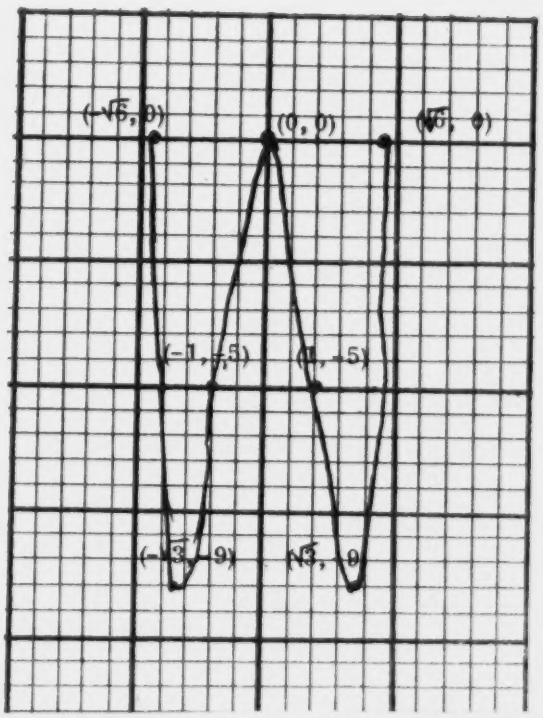
a. $\frac{2}{3}$ b. 0 c. $-\frac{4}{3}$ d. -2 e. -1

C.2 28. If the substitution $u = \sqrt{x + 1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x + 1}}$ is equivalent to:

a. $\int_1^2 \frac{du}{u^2 - 1}$ b. $\int_1^2 \frac{2du}{u^2 - 1}$

c. $\int_0^3 \frac{du}{(u - 1)\ (u + 1)}$ d. $2\int_1^3 \frac{du}{u(u^2 - 1)}$ e. $2\int_0^3 \frac{du}{u\ (u - 1)}$

KEY TO SAMPLE TEST ITEMS

1. $f^1(-2) = -7$
2. Tangent line $x = 1$; Normal line $y = 1$
3. b.
4. e.
5. a.
6. c.
7. e. Correct answer is $\frac{1}{(1 - x^2)^{3/2}}$
8. c.
9. a.
10. b.
11. b.
12. c.
13. Increasing $x < 0$; decreasing $x > 0$
14. a.
15. e.
16. c.
17. b.
18. a.
19. 

20. b.
21. a.
22. b.
23. c.
24. c.
25. a. $\frac{63}{32}$ b. $\frac{87}{32}$
26. d.
27. c.
28. b.